Why your COVD-19 exponential fits are wrong: words of caution and a lesson from catastrophe theory

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The following link [1] discusses the dangers of over-simplified models of epidemics/pandemics. What is surprising is that some of the simple models work quite well especially when sufficient data is accrued. Professional epidemiologists use very complex models and have relatively objective data. The problem with simple models such as the logistic model is that their parameters are obtained by fitting the observational data. These parameters are just that: fitting parameters. To be a physical model the simple mathematical models require much deeper analysis in terms of real objective testable theories so that they carry real predictive power.

A similar problem occurred in the 1970s when Renee Thom's Catastrophe Theory was in vogue , particularly as expounded by Chris Zeeman at the University of Warwick. Catastrophe Theory [2] often "obtained" extraordinarily convincing "fits" to experimental data on a variety of phenomena ranging from liquid-gas phase transitions to prison riots. The theory specialised in explaining how a smooth change in one state variable in a non-linear dynamical system defined on a manifold could lead to a discontinuous change in another variable. In most cases the mathematical theory was not corroborated in detail by physical theory. For example, with physical phase transitions it is usually the case that a few macroscopic degrees of freedom describe behaviour within a phase; an example is the thermodynamics of fluids. The simplest example of the equation of state for a fluid is Boyle's law:

$$PV = nRT \qquad (1)$$

where P, V, T are three state variables: pressure, volume and absolute temperature of a fluid. Here, $R=N_A k_B$ is the gas constant , N_A is Avogadro's constant and k_B is Boltzmann's constant. $n = N/N_A$ is the number of moles of the fluid.

Empirically, a better model introduced by van der Waal gives:

$$(P + a/V_M^{\,2})(V - nb) = nRT \qquad (2)$$

where again there are *three* state variables P, V, and T. $V_M = V/n$ is the molar volume (the volume of one mole of fluid). *There are two parameters a* and *b*.

Equation (2) provides a good fit to much experimental data by suitable empirical choices for *a* and *b*.

However, *a* and *b* represent *physical* quantities (and are therefore testable/measurable); for example, in van der Waal's theory *b* was related to the number of particles N and v the volume occupied by each particle ($V=N$v). The parameter $b = V_c/3$ where V_c is the molar volume at the critical point (P_c, V_c, T_c).

In Catastrophe Theory [2], the van der Waal equation and Maxwell's equal areas rule for the liquid gas phase transition are transformed into the standard form for the so-called cusp catastrophe three-variable model in a smooth abstract manifold. A continuous change in one of the state variables leads to a discontinuous jump in another variable.

Unfortunately, the underlying mathematics of catastrophe theory requires a *small* number of state variables and assumes the continuous existence of a potential field throughout the system. This is true within a particular equilibrium phase through the slaving of the ~ 10^{23} microscopic physical degrees of freedom to just three macro state variables. But a phase *transition,* as nowadays understood (through renormalisation group theory for example), involves a re-organisation and involvement of the full microscopic degrees of freedom. The variables a and b are pinned down by catastrophe theory but do not give the physical values correctly. References [2-3] discuss this failure in more detail. In short catastrophe theory often only generates an empirical theory with limited predictive power. *The reference* [1] *makes the same point about the predictive value and confidence levels of empirical curve fitting in simple epidemiological models*.

As a final thought, Enrico Fermi, a great theoretical physicist once said that given a new phenomenon any theorist worth his salt should be able to set up a simple theory that would be accurate to about 10%. Thereafter it might take tens of years of detailed complex study to produce a deeper predictive model with 1% accuracy. Horses for courses, indeed!

Acknowledgment

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References

1. Bruno Gonçalves,
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3. https://www.encyclopedia.com/science/encyclopedias-almanacs-transcripts-and-maps/rise-and-fall-catastrophe-theory (retrieved 2020).